GCE MARKING SCHEME

SUMMER 2016

Mathematics – S3
0985/01

INTRODUCTION

This marking scheme was used by WJEC for the Summer 2016 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

GCE Mathematics - S3
Summer 2016 Mark Scheme

<table>
<tr><th>Ques</th><th>Solution</th><th>Mark</th><th>Notes</th></tr>
<tr><td>1</td><td>The sample space and corresponding probabilities are as follows.

<table>
<tr><td>Sample</td><td>Max</td><td>Prob</td></tr>
<tr><td>2,2,2</td><td>2</td><td>1/20</td></tr>
<tr><td>2,2,10</td><td>10</td><td>6/20</td></tr>
<tr><td>2,2,50</td><td>50</td><td>3/20</td></tr>
<tr><td>2,10,10</td><td>10</td><td>3/20</td></tr>
<tr><td>2,10,50</td><td>50</td><td>6/20</td></tr>
<tr><td>10,10,50</td><td>50</td><td>1/20</td></tr>
</table>

$E(M) = 2 \times \frac{1}{20} + 10 \times \frac{9}{20} + 50 \times \frac{10}{20}$
-= 29.6 (p)</td><td>B3
B3

M1
A1</td><td>B3 for correct samples and max
B3 for correct probabilities
– 1 each error or omission</td></tr>
<tr><td>2(a)

(b)

(c)</td><td>$H_0 : \mu = 61; H_1 : \mu < 61$

$\sum x = 603.4$ si; $\sum x^2 = 36419.5$
UE of μ = 60.34
UE of $\sigma^2 = \frac{36419.5}{9} - \frac{603.4^2}{90}$
= 1.149 (431/375)

$\text{Test stat} = \frac{60.34 - 61}{\sqrt{\frac{1.149}{10}}}$
= – 1.947
DF = 9 si
Crit t value = 1.833
This result suggests that we should reject H_0, ie that the average miles per gallon is less than 61 because 1.947 > 1.833 oe</td><td>B1

B1B1
B1
M1
A1

M1A1

A1
B1
B1

B1
B1</td><td>

No working need be seen
M0 division by 10
Answer only no marks

M0 for no working
Note that p-value = 0.0417

FT the conclusion
No FT for reason if z-value used</td></tr>
</table>

Ques	Solution	Mark	Notes
3(a)	$\hat{p} = \frac{44}{80} = 0.55$ si	B1	
	ESE = $\sqrt{\frac{0.55 \times 0.45}{80}}$ (= 0.0556..) si	M1A1	M1A0 if √ omitted
	90% confidence limits are $0.55 \pm 1.645 \times 0.0556..$	M1A1	M1 correct form, A1 correct z
	giving [0.459,0.641]	A1	
(b)(i)	$\hat{q} = \frac{0.555 + 0.705}{2} = 0.63$	B1	
	Games won = 0.63 × 100 = 63	B1	
(ii)	$0.705 - 0.555 = 2 \times z\sqrt{\frac{0.63 \times 0.37}{100}}$ or equiv	M1A1	
	z = 1.55	A1	
	Prob from tables = 0.0606 (0.9394)	A1	
	Confidence level = 88%	A1	
4(a)	$H_0 : \mu_A = \mu_B ; H_1 : \mu_A \neq \mu_B$	B1	
(b)	$\bar{x} = 251.6; \bar{y} = 251.4$ or $\bar{x} - \bar{y} = 0.2$	B1	
	$s_x^2 = \frac{5064256}{79} - \frac{20128^2}{79 \times 80} = 0.648...(256/395)$	M1A1	
	$s_y^2 = \frac{5056222}{79} - \frac{20112^2}{79 \times 80} = 0.825...(326/395)$	A1	
	[Accept division by 80 giving 0.64 and 0.815..]		
	SE = $\sqrt{\frac{0.648..}{80} + \frac{0.825..}{80}}$ = 0.135.. (0.1348…)	M1A1	
	$z = \frac{251.6 - 251.4}{0.135..}$	m1	
	=1.47 or 1.48	A1	
	Prob from tables = 0.071or 0.069	A1	
	p-value = 0.14	B1	FT from line above
	Insufficient evidence to reject H_0	B1	FT the p-value
(c)	The CLT allows us to assume that the distributions of the sample means are (approximately) normal	B1	

Ques	Solution	Mark	Notes
5(a)	$\sum x = 210, \sum x^2 = 9100,$ $\sum y = 1286, \sum xy = 48730$	**B2**	Minus 1 each error
	$S_{xy} = 48730 - 210 \times 1286/6 = 3720$	**B1**	
	$S_{xx} = 9100 - 210^2/6 = 1750$	**B1**	
	$b = \dfrac{3720}{1750} = 2.13$ (372/175)	**M1A1**	M0 no working
	$a = \dfrac{1286 - 2.13 \times 210}{6} = 140$ (2099/15)	**M1A1**	M0 no working
(b)(i)	SE of $b = \dfrac{1.5}{\sqrt{1750}}$ (0.0358..)	**M1A1**	FT from (a)
	95% confidence limits are 2.1257.. ± 1.96 × 0.0358.. [2.06, 2.20]	**m1A1** **A1**	
(ii)	$x_0 = 35$	**B1**	
	Because the SE of y or the width of the interval is minimum when $x_0 = \bar{x}$	**B1**	

Ques	Solution	Mark	Notes
6(a)(i)	$E(\bar{X}) = \dfrac{\sum_{i=1}^{n} E(X_i)}{n}$	M1	
	$= \dfrac{n\mu}{n} = \mu$	A1	
	(Therefore $\bar{X}$ is an unbiased estimator)		
(ii)	$\text{Var}(\bar{X}) = \dfrac{\sum_{i=1}^{n} \text{Var}(X_i)}{n^2}$	M1	
	$= \dfrac{n\sigma^2}{n^2} = \dfrac{\sigma^2}{n}$	A1	
	SE of $\bar{X} = \dfrac{\sigma}{\sqrt{n}}$		
(b)(i)	$\text{Var}(X_i) = E(X_i^2) - [E(X_i)]^2$	M1	
	$\sigma^2 = E(X_i^2) - \mu^2$	A1	
	$E(X_i^2) = \mu^2 + \sigma^2$		
(ii)	$E(S^2) = \dfrac{\sum_{i=1}^{n} E(X_i^2) - nE(\bar{X}^2)}{n-1}$	M1	
	$= \dfrac{n(\mu^2+\sigma^2) - n\left(\mu^2 + \dfrac{\sigma^2}{n}\right)}{n-1}$	A1A1	
	$= \sigma^2$		
(c)	$\text{Var}(S) = E(S^2) - [E(S)]^2$	M1	
	$[E(S)]^2 = \sigma^2 - \text{Var}(S)$	M1	
	$< \sigma^2$ (since $\text{Var}(S) > 0$)	A1	
	Therefore $E(S) < \sigma$ so $E(S) \neq \sigma$	A1	FT above line if both M marks awarded
	(Therefore S is not an unbiased estimator for σ)		